PARCC Mathematics Performance Tasks

TWO NUMBERS

Overview

At a Glance

In this game, students roll 2 number cubes and flip numeral cards (0–9) based on what they see on the cubes. The object is to turn all cards face down.

Grade Level

Kindergarten (This task can also be used in Grade 1)

Task Format

- Partner game (2 students); modeled whole class
- Task could be repeated over a series of 4–5 days at teacher's discretion

Materials Needed

For each pair of students

- 2 dot cubes or 2 number cubes (1 through 6)
  Recommended: Initially, use cubes with dots  , not numerals  .
  Later, use cubes that have numerals to develop and assess different skills.
- 2 sets of 0–10 cards (one for each student).

For the teacher

- Observation Checklist

Prerequisite Concepts/Skills

When the use of this task is focused on addition/subtraction, it requires:

- Recognition of numerals 0–10 and
- Ability to count images or objects reliably through 10.

This instructional task is designed to help teachers recognize gaps in those skills and help learners acquire the skills. Adjustments written into the instructions allow this task to be used with early learners from roughly 4½ years of age through grade 2 to build these skills.

Content Standards Addressed in This Task

K.OA.A.5 Fluently add and subtract within 5.

K.CC.B.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Extensions and Elaborations

1.OA.C.6 [Demonstrate] fluency for addition and subtraction within 10.

Standards for Mathematical Practice Embedded in This Task

MP1 Make sense of problems and persevere in solving them.

MP7 Look for and make use of structure.

GET READY: Familiarize Yourself with the Mathematics

This task assesses students' understanding of how to add and subtract fluently within 5 (K.OA.A.5). Embedded within this task is standard K.CC.B.5, which prompts students to answer "how many" questions about a number of objects. In the case of this task, the objects are dots on a dot cube. This task is divided into two parts, which increase in difficulty:

Part 1: Students play the game using cubes with dots on them, allowing them a visual support to count quantities as needed.

Part 2: Students progress to using numeral cubes only. Part 2 provides less scaffolding and allows you to observe standard K.OA.A.5 with numerals only.

Each stage is its own subtask. Therefore, you may choose to begin all students on Part 1 and, depending upon your observations, move some students to Part 2, or you may return to implement Part 2 at a later date with your whole class, entirely at your discretion.

Adding and Subtracting Fluently within 5

Many mathematical understandings and skills underlie fluency at addition and subtraction within 5. Faced with the unfinished written equation 3 + 2 = ___ , one student might understand that addition means "putting together" or "adding to" (see K.OA), and that student would likely count to figure out the answer. A second student might know perfectly well what having three and getting two more means and fluently respond with "five," but might not be able to handle the on-paper computation because he or she either doesn't recognize the numerals or isn't sure what that equation is saying. A third student might correctly and without hesitation say "five!" and write 5 in the blank, but have that as a memorized fact, without the more general understanding of addition as putting together or adding to. True *fluency*, standard K.OA.A.5, requires it all: recognizing the numerals and the written representation, understanding that it calls for addition, understanding what addition *means*, and being able to retrieve the sum without hesitation.

Though full fluency does require numeral recognition, it is important to distinguish what a student knows and understands from what the student can easily read and write. For example, letter/numeral reversal in writing is not a mathematical weakness but a separate (essentially developmental) matter. Likewise, a student whose reading lags behind oral understanding (the second student in the description above) is quite different from a student who can read perfectly but does not understand the mathematics or does not know the mathematical facts. Effective instruction depends on knowing what a child does know as well as recognizing what the child still needs to learn.

Subitizing. Another essential aspect underlying students understanding of number and quantity is subitizing. Subitizing is instantly seeing how many for small collections of quantities. More specifically, there are two types of subitizing: *perceptual subitizing* and *conceptual subitizing* (Clements, 1999). "Perceptual subitizing is the closest to the original definition of subitizing: recognizing a number without using other mathematical processes" (p. 40). An example of perceptual subitizing is holding up a group of fingers showing small numbers such as  (2). In this instance, the student should not need to count "one, two" but instead instantly know that there are two. Conceptual subitizing builds upon this first type where students use their ability to instantly recognize smaller quantities ≤ 5 and apply this understanding to larger numbers. For example, a student may see the arrangement of 6 () on a dot

cube and *perceptually* subitize two groups of three and simply know that the quantity is equal to 6. The 6 is a result of *conceptual* subitizing: the 6, itself, was not seen but instantly known from the two 3s.

It is also important to note that the game that they will play in this task sets a significant cognitive challenge for young students by requiring them to make a strategic decision about what to do with the two numbers (e.g., 2 and 3) they see: use them as two separate numbers, add them (e.g., to get 5), or subtract them (e.g., to get 1).

Standards for Mathematical Practice

When students are playing this game, they must remember the goal: turning all the cards face down. To achieve that goal, students must decide what to do with the numbers they roll: use them as is, add, or subtract. This is engagement in *MP1*: *Make sense of problems and persevere in solving them.* This task also gives students useful experience with "looking closely to discern a pattern or structure," which is *MP7: Look for and make use of structure* (NGA & CCSSO, 2010). As students learn to count to tell the number of objects, they gain practice with the ordered list of number words. Students also begin to experience the structure embedded within adding and subtracting numbers. For example, a student may roll a and and think, "I know that 2 + 2 is 4, so 2 + 3 must be 5." This task begins to build the foundation for making use of structure that will eventually become critical for students in understanding operations with larger numbers.

GET SET: Preparing to Introduce the Task

This is a partner game, to be first modeled by the teacher, but then played in pairs. Throughout this document, when specific language is suggested, it is shown in *italics.*

1. Gather the materials listed on page 1. Each pair of students should receive 1 set of numeral cards. If you would like students to focus only on addition, give each pair numeral cards 1 to 10. If you would like students to focus on both addition and subtraction, give each player a 0 as well (that is, numeral cards 0 to 10). Players will share the two cubes (with dots or numerals).
2. Pair students ahead of time. You may choose to switch students' partners for each subtask or at your discretion.
3. The initial introduction may be modeled with a small group or possibly the whole class.
4. If you already know that your children easily recognize all the numerals from 0 through 10 (including the look-alike 6 and 9) and can reliably set them in order, skip directly to Introducing the Task. Otherwise, start with the Setting up the Task instructions below.

Setting Up the Task

Hand each student a set of 1–10 cards. (If you are offering a subtraction option, use 0–10 cards.)

- *Would you check to see if you have all the numbers from 0 (or 1) to 10?*
- (If the students don't naturally organize them in some way, you might *choose* to prompt this arrangement) *Arrange all of your number cards in order by putting them in two rows.*

Now skip to step 3 in the Introducing the Task section below.

Introducing the Task

1. Hand each player a set of cards (either 1–10 or 0–10). Use 0 only if you decide students are ready for the option of subtraction.
2. To each player: *Place your cards on the table face up. Check to see if you have all the numbers. Arrange your numbers in order.* (It can be useful, and space-saving, to arrange them in two rows.)
3. To Player 1: *You'll start. Roll your dice.* Our example here will assume the roll was  and . *What numbers did you roll?* Let student name the two numbers.
4. *Now you get to turn over one or two cards. You can turn over the numbers you see OR you can add the two numbers and turn over the total OR* (when they are using subtraction option) *you can subtract one number from the other and turn over the difference. Your goal is to get all of your cards turned over.*
5. To Player 2: *Now it's your turn to do the same thing.*
6. To both players: *If you can't turn over any numbers, that's okay; just let your partner take a turn.*

Help students get started. They can play on their own, getting practice with both the arithmetic facts and with the strategy, but if you watch, you can learn a great deal about how they are solving the problems. Often, they prefer to add if they can, even if they would be able to turn more cards over by not adding! And often, students spontaneously help each other. Encourage "oh I see something you can do" rather than "do that." Of course, if that help is not enough, let the helper say more.

Preparing to Gather Observation Data and Determine Next Steps in Instruction

As students engage in the task, the notes in the next section will help you identify students' current strengths and possible next steps for instruction. As you observe, use whichever form of the Observation Checklist that best helps you record your observations of students and other relevant evidence as you see it: Individual, Partner, or Class. These varied forms, available at the end of this document and in a separate MS Excel file, are intended to give you a choice about how to collect notes on your students and determine possible next steps for instruction.

Addressing Student Misconceptions/Errors

Students who are used to playing with number cubes may be so used to adding them (or just using the numbers as they see them, without any adding or subtracting) that, despite your instructions, they may forget to subtract. That does not necessarily mean that they can't subtract. If students can't make any of their remaining numbers with addition, and miss an opportunity to make one of their remaining numbers with subtraction, remind them that they have the choice.

If students seem unable to subtract, or seem uncertain about what that means, you can point to the cube with the smaller number of dots and say something like, "See what's left if you cover up that many dots on the other cube!" Then help students, if necessary, to cover up the right number of dots and see what remains. You can also use a dot card and numeral cube to see if students are counting on or subitizing. This is another way to differentiate the task.

When students use cubes with dots rather than numerals, they may fluently add within 5 (the K standard) just by subitizing, rather than by knowing the fact. That is, a student may easily see the 3 on both dot cubes ( and) without consciously knowing the answer to What is 1 plus 2? You can check that with a question, or by playing the game with cubes that use numerals, rather than dots. Starting with dot-cubes helps the beginners, especially with larger numbers, and lets you see whether they can

subitize, and whether they use that for counting on. Subitizing is the ability to see a small amount of objects and know instantly how many there are without counting. It is what tells you what number you roll on six-sided dice. It is a fundamental skill in the development of students' understanding of number.

Extensions and Elaborations

Even though the K standard is the sums of numbers ≤5, some students will comfortably know sums greater than 5. Some may even be familiar enough with dot cubes that they know  and  and wonder what to do with them if they are rolled, because the highest card to flip is 10. If they don't think about flipping 5 and/or 6 (or can't, because those are already used), you might remind them that subtraction is an option. You could also decide to include numeral cards 11 and 12 to allow for this extension. Because children love playing the game, they pay attention to the numbers they throw. It is not unusual to hear children say "Oh, I got a five and a six *again*!" and, after not many occurrences, know exactly what number that makes. Children also say things like "I got a three and a four. That's what *you* need!"

GO: Carry Out the Task

This game is not complex, but students in kindergarten vary considerably in what they know about number, counting, addition, and subtraction. They also vary in how well they can switch between attending to the numbers on the cubes and numbers on the cards, and how flexibly they can *switch* between adding and subtracting to meet a goal. There is a great deal to learn as you observe students play, especially as they play the first few times.

Part 1: Introduce the game with dot cubes 1 to 6 (dots, not numerals, on them)

Task steps	Keep in Mind	Observations of Students
1. Hand cards to both students and ask each to set them out face-up to check to see if they have all the numbers from 1–10. If your students are ready for subtraction, use number cards 0–10. SAY to BOTH PLAYERS: *Place your cards on the table face up. Would you check to see if you have all the numbers from 1 to 10? Arrange all of your number cards in order by placing them in two rows.*	• Some children will not recognize all the numerals correctly. Many children read through them but do not spontaneously sort them in order to check that they have them all and that there are no repeats. • For a child who does not know all the numerals, eliminate 0, put down the 1, then point to the 1 and ask if the child knows that number. Then say "What number comes after 1?" When the child says "two," you set out the card with the 2 on it, and continue, setting out two rows, 1–5 and 6–10. Rehearse once, with the child pointing to each card as he or she recites the numbers in order.	A. Student does not recognize all the numerals. (Record which are missed). B. Student reads the numerals correctly, but is not systematic about knowing if they are all there and all different. C. Student arranges *some* of the numbers in order, but makes *at least* one error in the placement of a number. D. Student correctly arranges the numbers in order.
2. *Explain to students that the goal of the game is to try to turn all of the number cards face-down.* Hand the dot or number cubes to Player 1. Emphasize that players should say the numbers rolled aloud. EXPLAIN to BOTH PLAYERS: *When it's your turn, roll the two cubes and then say the numbers that you rolled out*	• When you are playing with **dot cubes**, pay attention to which numbers of dots the student immediately recognizes and which s/he appears to need to count. • If the student counts the collections of **dots**, does s/he do so correctly matching each dot with a single counting word (one to one) and recognize that the last number is the number of dots on the cube? • It is not unusual for children at this age to	E. Student recognizes  ,  , and  readily. Student counts for at least  and  and sometimes makes errors. F. Student needs to count for larger numbers of dots, counts correctly, but does not yet retain that pattern the next time that pattern is encountered. G. Student recognizes all of the numbers of dots readily. H. (If using numeral cubes) Student

loud. SAY to PLAYER 1: *Roll the two cubes. What numbers did you roll?*	count to find out how many dots are on  ,  , and  , but some will recognize even these numbers readily. • It is *very* unusual for children to need to count to find out how many dots are on  ,  , and  . Children may need counting strategies for *operations* (addition or subtraction) involving these numbers, but if they seem to be counting just to know how many dots are on those cubes, probe to see whether they *need* to or are just doing it by habit or for fun. • When you are playing with **numeral cubes**, *less* fluency likely suggests that more work on numeral recognition is needed. It may also suggest more need for models and representations to connect numerals to amounts in order to build mental images.	recognizes all numerals.
3. SAY to BOTH PLAYERS: *Now, you have two (or three) choices. You can turn down the exact numbers you see; you can add the two numbers and turn over the sum; or* (optional) *you can subtract one number from the other and turn over the difference. For example, if you roll 4 and 3, you may turn over 4 and 3, or 7 by adding, or 1 by subtracting.* Tell players that they should explain their move aloud to their partners. For example, a student may say, "I rolled 4 and 3. I turned over 7 because 4 plus 3 equals 7."	• Keep in mind that you may first decide to play this game offering students only two choices: turning over the numbers that were rolled or adding those numbers and turning over the sum. As students become more skilled with the game and with subtraction, include that choice as well. • You may notice some students focus on adding the numbers and forget that they can turn over the exact numbers that they see (instead of losing a turn) or they can subtract one number from another. Remind them if necessary. At this age, players may spontaneously help their	I. Student counts starting at 1. J. Student recognizes the number of dots on one cube and continues that count while pointing to the dots on the other cube. K. Student seems to know many totals without counting. Note which totals the student does know, and where the student relies on counting. L. Student *fluently* adds within 5 when using numeral cubes. M. Student performs additions whose sum is greater than 5, using strategies like counting, drawing, decomposing or adjustment from other known sums.

SAY to BOTH PLAYERS:

If you can't turn over any numbers, that's okay, you just let your partner take a turn.

Now, allow Player 1 a chance to make his or her move.

opponents that way.

- How do students add? Do they…
  - count all the dots, starting at 1?
  - recognize the number on one cube and then count on from there to add the number on the second cube?
  - recognize the total without obvious counting? This may be from visual recognition (subitizing) of pairs like   or even   or by subitizing the individual numbers in pairs like   and knowing the addition fact that tells them this number is 7. Instant visual recognition of numbers greater than 5 generally requires some kind of visual chunking and then either memory for that configuration or a number fact. For example, we recognize as 6 either because we have learned to recognize that configuration or because we see 3 + 3 and know that fact. We do not, in general, recognize six randomly arranged dots as 6 without seeing and adding the numbers in smaller clusters of them.
- A student who generally recognizes totals up to 5 probably does meet the addition part of standard K.OA.A.5, but watch the child play the game with cubes that have numerals (not dots) to be sure. It sometimes happens that children using dot cubes will subitize   as 5 without knowing that the image represents the

N. Student appears fluent with some ≥ 5 additions. (Note which ones.)

	question "What is 3 + 2?" • Though the K standards call for **fluency** only within 5, they also look for addition and subtraction within 10. Watch for **fluency** in your students when they use numeral cubes and roll numbers with sums ≤5.	
	• Do students remember the goal and know when it is useful to subtract? – It is common, even for children who do know what subtraction means and how to do it, to forget to use it. – In fact, it is common for young children to look at two dice, find the sum, and forget even that they may consider the dice independently and turn each of the numbers they actually see. – The hope here is that students are starting to make sense of what addition and subtraction are and begin to use models to represent it.	O. Student uses both numbers or the sum, but does not independently use subtraction. P. Student readily recognizes the option of subtraction (e.g., the option to turn over 1 when the pair is ). Q. Student recognizes that the only way to turn over 0 is to get two numbers that are the same (e.g., ).
	• How do students subtract? – Young students vary in their strategies for subtraction. Some understand the connection between addition and subtraction and readily use the fact that they can look at the smaller number and figure out what to add to it to get the bigger number; some (when using objects) choose the larger set and *cover* as many objects as indicated by the smaller set and then count the remainder; some count backwards;	R. Student works out subtraction using some strategy but does not seem to know the facts fluently. S. Student meets fluency standard in (+) and (-).

	some know facts. – With numbers ≤ 5, some students get answers quickly enough that it is hard to tell which strategy they used. Often, they do not know themselves how they figured out the answer. Often in those cases, they say things like "I just knew it."	
4. Once Player 1 finishes a turn, pass the dot cubes to Player 2. Player 2 should repeat Steps 2 and 3. Continue to play until both players have turned over all of the cards.		

Part 2: Play the game with numeral cubes 1 to 6 (not dots).

Without dots to count, the demands on a student's thinking are different. A student may still count or count on, but must do so without the aid of *given* visual objects to count (perhaps by replacing them with fingers or by drawing them). A student who readily reported a total when using dots can now show that the success is not from subitizing small numbers or from fast-counting bigger ones, but from knowledge of sums and differences.

Task steps	Keep in Mind	Observations of Students
1. Repeat Steps 1—4 from Part 1 (above), now playing the game with two numeral cubes (not dot cubes). Now that students are playing with numeral cubes, be sure to pay attention to… • which numerals students readily recognize. • which sums student readily know. • which differences students readily know.	• With number cubes, most sums are ≤10, but two sums are greater (5 + 6 or 6 + 5 and 6 + 6). • Subtractions of like numbers, such as 6 – 6 and 5 – 5, are important in this game, as they are the only ways of getting 0. Similarly, subtraction of consecutive numbers is the only way to get 1.	T. List sums (addition) that student knows fluently when presented only with pairs of numerals. U. List differences (subtraction) that student knows fluently when presented only with pairs of numerals.

OBSERVATION CHECKLIST

ASSESSING STUDENT UNDERSTANDING: TWO NUMBERS

Use this page to record individual student observations. Use the letters to notate each event as you see it unfold. This record is intended to help you plan next steps in your instruction for your students.

Student Name	Observation of Students	Possible Individual Student Observations
		COUNTING and FLUENCY A. Student does not recognize all the numerals. (Record which are missed). B. Student reads the numerals correctly, but is not systematic about knowing if they are all there and all different. C. Student arranges *some* of the numbers in order, but makes *at least* one error in the placement of a number. D. Student correctly arranges the numbers in order. **MAKING MEANING** E. Student recognizes , , and readily. Student counts for at least and  and sometimes makes errors. F. Student needs to count for larger numbers of dots, counts correctly, but does not yet retain that pattern the next time that pattern is encountered. G. Student recognizes all of the numbers readily. H. (If using numeral cubes) Student recognizes all numerals. **COUNTING and FLUENCY** I. Student counts starting at 1. J. Student recognizes the number of dots on one cube and continues that count while pointing to the dots on the other cube.	K. Student seems to know many totals without counting. Note which totals the student does know, and where the student relies on counting. L. Student *fluently* adds within 5 when using numeral cubes. M. Student performs additions whose sum is greater than 5, using strategies like counting, drawing, decomposing or adjustment from other known sums. N. Student appears fluent with some ≥5 additions. (Note which ones.) **STRATEGIES** O. Student uses both numbers or their sum, but does not independently use subtraction. P. Student readily recognizes the option of subtraction (e.g., the option to turn over 1 when the pair is ). Q. Student recognizes that the only way to turn over 0 is to get two numbers that are the same (e.g., 3 3). **FLUENCY** R. Student works out subtraction using some strategy but does not seem to know the facts fluently. S. Student meets fluency standard in (+) and (-). T. List sums (addition) that student knows fluently when presented only with pairs of numerals. U. List differences (subtraction) that student knows fluently when presented only with pairs of numerals.

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