

## MATH-LIBS: SOLVING TWO-STEP WORD PROBLEMS

Overview	
<b>At a Glance</b>	In this Mad-Libs-like 2-player game, students choose objects and numbers according to given criteria and solve word problems based on their choices.
<b>Grade Level</b>	Grade 2
<b>Task Format</b>	<ul style="list-style-type: none"> <li>• Partner game (2 students); modeled whole class or small group</li> <li>• Each subtask may be repeated over 2–3 days; 15–20 min per day</li> </ul>
<b>Materials Needed</b>	<p><i>For each pair of students</i></p> <ul style="list-style-type: none"> <li>• 1 writing surface, such as a clipboard or a table</li> <li>• 2 pencils (one per student)</li> <li>• Part 1: 1 set of Story Cards—Set A (template provided)</li> <li>• Part 2: 1 set of Story Cards—Set B &amp; Set C (templates provided)</li> <li>• <i>Extension/Elaboration</i>: 1 set of Story Cards—Set D (template provided)</li> </ul> <p><i>For the teacher</i></p> <ul style="list-style-type: none"> <li>• Physical objects, such as counters or base-10 blocks, available to pairs of students upon request or teacher decision</li> <li>• Observation Checklist (template provided)</li> </ul>
<b>Prerequisite Concepts/Skills</b>	<ul style="list-style-type: none"> <li>• Experience in solving one-step addition and subtraction word problems within 20 involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions</li> <li>• Familiarity in representing a solution using objects, drawings, or equations</li> <li>• Understanding that the two digits of a two-digit number represent amounts of tens and ones</li> <li>• Composing or decomposing a ten from 10 ones</li> <li>• Familiarity with using various addition and subtraction strategies, including counting on, composing a ten, decomposing a number leading to a ten, and using the relationship between addition and subtraction</li> </ul>

### Content Standards Addressed in This Task

**2.OA.A.1** Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).

**Note:** While 2.OA.A.1 calls for solving one- and two-step word problems, this task addresses *only* two-step word problems.

Extension(s) and Elaboration(s)

**3.OA.D.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

### Standards for Mathematical Practice Embedded in This Task

**MP1** Make sense of problems and persevere in solving them.

**MP2** Reason abstractly and quantitatively.

## GET READY: Familiarize Yourself with the Mathematics

This task's purpose is for students to demonstrate their understanding of *how* to solve various addition and subtraction situations involving two steps. The two parts of this task increase in difficulty.

The chart below gives you a quick introduction to the different types and subtypes of word problems for grades K–2.

ADD/SUBTRACT SITUATION TYPE	SUBTYPES	LEVEL OF DIFFICULTY
Add To/Take From	Result Unknown Change Unknown Start Unknown	Easy Medium or Difficult Difficult
Put Together/Take Apart	Total Unknown Both Addends Unknown Addend Unknown	Easy Medium or Difficult Medium
Compare	Difference Unknown Bigger Unknown Smaller Unknown	Medium or Difficult (depending on language used)

Adapted from NCTM (2011)

**Note:** A description of these types and subtypes of situations and levels of difficulty is included below.

**Part 1** focuses on having students solve a combination of two-step addition and subtraction situations involving most types and subtypes of word problems with single-digit and teen totals  $\leq 18$ .

**Part 2** has students facing problems of similar difficulty, but with totals from 20 to 100 when using Story Cards—Set B. Students advance to solving more difficult subtypes of word problems with totals from 20 to 100 when using Story Cards—Set C.

Each part is its own task. You can begin all students on Part 1 and, depending upon your observations, move some students to Part 2 right away, or you might return to Part 2 at a later date with your whole class. Use this tool in whatever way helps you and your students most.

The Story Card sets (A, B, C, and D) are built in a progression. Story Cards—Set A, used in Part 1, includes two-step word problems (levels 1 to 3) with single-digit numbers and teen totals  $\leq 18$ . Story Cards—Set B, used in Part 2, progresses towards solving two-step word problems (levels 1 to 3) with totals from 20 through 100. Story Cards—Set C moves students toward applying their understandings to two-step word problems at levels 3, 4 and 5 with totals from 20 through 100. This set includes Compare problems involving misleading language suggesting the wrong operation. Set D is for extension only and asks students to write an equation that represents the story. You might choose to have students also solve that equation, based on individual readiness.

### Types of Addition and Subtraction Situations

Understanding how to “mathematize” and model addition and subtraction situations using objects, drawings, and equations becomes a foundation for later algebraic problem solving (NCTM, 2011, p. 44). When given a story problem, students first make meaning of the situation and then apply this understanding to build a representation of the problem that helps them choose or develop a strategy for finding a solution. The representation may take the form of objects, drawings, or written equations.

Standard 2.OA.A.1 calls for students to become proficient in solving three types of addition and subtraction situations: Add To/Take From, Put Together/Take Apart, and Compare. For each of these, there are three subtypes (explained below) depending on which quantities are known. Gaining skill at solving these types of problems requires ample experience. This task presents two-step word problems, so each “step” of the problem can (and does) provide experience with one of the given subtypes.

Students should solve these problems using the representation that best suits *where* they are: one may be ready to solve a problem with a written equation; another may be just as capable of solving the problem but does it by modeling the problem with objects. Variations among students are expected and the type of representation they use is worth noting. It is a goal for grade-2 students to be able to represent these situations using drawings and equations, so aim to help them move towards that goal.

**Add To/Take From.** One type of addition or subtraction situation involves adding objects to or taking objects from a collection (NGA & CCSSO, 2010). These have also been called “Change Plus” or “Change Minus” problems (Cross, Woods, & Schweingruber, 2009; NCTM, 2009). These situations involve three quantities:  $A + B = C$  or  $A - B = C$ , in which we have a starting quantity (A), an amount by which this quantity changes (B), and the resulting quantity (C). The most familiar problem type—Mary has 4 crayons and Derek gives her 3 more; how many does she now have?—is the Result Unknown, where A and B are known and the student must determine C, the sum. Altering the action results in Start Unknown  $\square + B = C$ , or Change Unknown  $A + \square = C$  problems, and more experience with these important variations increases children’s familiarity with them, and their flexibility and ease in problem solving. Table 1 below (from NGA & CCSSO, 2010, p. 88) shows examples of these three subtypes. Keep in mind that standard 2.OA.A.1 calls for unknowns in all positions. Therefore, you should find that by

grade 2 the majority of your students are ready to focus more on Change Unknown and Start Unknown situations.

Table 1. Add to/Take From Situations

	Result Unknown	Change Unknown	Start Unknown
<b>Add to</b>	<p>A bunnies sat on the grass. B more bunnies hopped there. How many bunnies are on the grass now?</p> $A + B = \square$ <p>(e.g., Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? <math>2 + 3 = \square</math>)</p>	<p>A bunnies were sitting on the grass. Some more bunnies hopped there. Then there were C bunnies. How many bunnies hopped over to the A bunnies?</p> $A + \square = C$ <p>(e.g., Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? <math>2 + \square = 5</math>)</p>	<p>Some bunnies were sitting on the grass. B more bunnies hopped there. Then there were C bunnies. How many bunnies were on the grass before?</p> $\square + B = C$ <p>(e.g., Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? <math>\square + 3 = 5</math>)</p>
<b>Take from</b>	<p>C apples were on the table. I ate B apples. How many apples are on the table now?</p> $C - B = \square$ <p>(e.g., Five apples were on the table. I ate two apples. How many apples are on the table now? <math>5 - 2 = \square</math>)</p>	<p>C apples were on the table. I ate some apples. Then there were A apples. How many apples did I eat?</p> $C - \square = A$ <p>(e.g., Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? <math>5 - \square = 3</math>)</p>	<p>Some apples were on the table. I ate B apples. Then there were A apples. How many apples were on the table before?</p> $\square - B = A$ <p>(e.g., Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? <math>\square - 2 = 3</math>)</p>

**Put Together/Take Apart.** A second context for addition and subtraction involves combining separate collections of objects or splitting a single collection into two parts (NGA & CCSSO, 2010). In “Put Together” situations, two known parts, (A) and (B), combine to make a larger one (C) whose size we must figure out (Cross, Woods, & Schweingruber, 2009). Again, students may be solving to find any of these numbers. However, in all cases, students are using the addition equation  $A + B = C$  as a foundation and making appropriate changes to the equation according to the context of the word problem (e.g.,  $A + \square = C$ ;  $\square + B = C$ ; or  $C - \square = B$  or A). Take-Apart situations work in the reverse. In these problems, the larger set’s size (C) is known, but one or both of the parts (A or B) is not. Here students have to determine an unknown part or find all of the possible ways to break the sum into two parts when both are unknown. Table 2 (NGA & CCSSO, 2010) shows examples of these three subtypes.



Table 2. Put Together/ Take Apart Situations

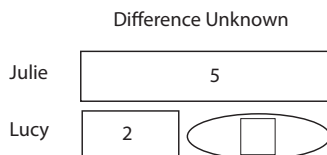
	Total Unknown	Addend Unknown	Both Addends Unknown
<b>Put Together/ Take Apart</b>	<p>A red apples and B green apples are on the table. How many apples are on the table?</p> $A + B = \square$ <p>(e.g., Three red apples and 2 green apples are on the table. How many apples are on the table? <math>3 + 2 = \square</math>)</p>	<p>C apples are on the table. A are red and the rest are green. How many apples are green?</p> $A + \square = C$ $C - A = \square$ <p>(e.g., Five apples are on the table. Three are red and the rest are green. How many apples are green? <math>3 + \square = 5</math> or <math>5 - 3 = \square</math>)</p>	<p>Grandma has C flowers. How many can she put in her red vase and how many in her blue vase?</p> $\square + \square = C$ <p>(e.g., Grandma has five flowers. How many can she put in her red vase and how many in her blue vase?)</p> $5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$

**Compare.** A third context for addition and subtraction involves comparing two quantities (NGA & CCSSO, 2010), finding the difference between a larger quantity (C) and a smaller quantity (A or B). This difference, the third addend, is not present in the given situation. Students may see this difference as the “extra leftovers in the bigger quantity or [as] the amount the smaller quantity needs to gain to be the same as the bigger quantity” (NCTM, 2009, p. 41). Table 3 (NGA & CCSSO, 2010, p. 88) shows examples of these three subtypes.

Table 3. Comparison Situations

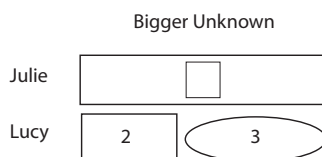
	Difference Unknown	Bigger Unknown	Smaller Unknown
<b>Compare</b>	<p>“How many more?” version:</p> <p>Lucy has A apples. Julie has C apples. How many more apples does Julie have than Lucy?</p> $A + \square = C$ <p>(e.g., Lucy has 2 apples. Julie has 5 apples. How many more apples does Julie have than Lucy? <math>2 + \square = 5</math>)</p>	<p>“More” version:</p> <p>Julie has B more apples than Lucy. Lucy has A apples. How many apples does Julie have?</p> $A + B = \square$ <p>(e.g., Julie has 3 more apples than Lucy. Lucy has 2 apples. How many apples does Julie have? <math>2 + 3 = \square</math>)</p>	<p>“Fewer” version:</p> <p>Lucy has B fewer apples than Julie. Julie has C apples. How many apples does Lucy have?</p> $C - B = \square$ <p>(e.g., Lucy has 3 fewer apples than Julie. Julie has 5 apples. How many apples does Lucy have? <math>5 - 3 = \square</math>)</p>
	<p>“How many fewer?” version:</p> <p>Lucy has A apples. Julie has C apples. How many fewer apples does Lucy have than Julie?</p> $C - A = \square$ <p>(e.g., Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? <math>5 - 2 = \square</math>)</p>	<p>“Fewer” version with misleading language:</p> <p>Lucy has B fewer apples than Julie. Lucy has A apples. How many apples does Julie have?</p> $B + A = \square$ <p>(e.g., Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? <math>3 + 2 = \square</math>)</p>	<p>“More” version with misleading language:</p> <p>Julie has B more apples than Lucy. Julie has C apples. How many apples does Lucy have?</p> $\square + B = C$ <p>(e.g., Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? <math>\square + 3 = 5</math>)</p>

In the Difference Unknown situation, a student may think of Lucy's and Julie's apples using this type of representation.

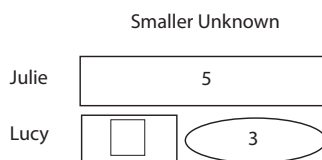


A student may then use *either addition or subtraction* correctly to solve this problem. For example, a student may think, “I know that Julie has 5 apples, so if I subtract 2, the number of apples Lucy has, I will know how many more apples Julie has than Lucy (or how many fewer apples Lucy has than Julie).” Or, a student may think  $2 + \square = 5$  and explain, “If Lucy has 2 apples and Julie has 5 apples, then she’d need 3 more apples to have the same as Julie.” Either of these ways of thinking makes sense given the situation.

In a “Bigger Unknown” problem, using the apples example, we know that Julie has 3 more apples than Lucy (or Lucy has 3 fewer apples than Julie), but don’t yet know how many apples Julie has.



In a “Smaller Unknown” problem, the unknown number changes its place in the equation: the number of apples Lucy has is unknown.



In the past, comparison situations have generally been treated as if only a subtraction equation was legitimate. This treatment of comparison situations is too narrow. Remain aware of this as you observe your students, because students think about these situations differently.

Students need multiple opportunities to hear and say words “more” and “fewer” (or “less”) when solving comparison problems. According to NCTM (2009), “practicing saying the comparison both ways is helpful in building the linguistic competence for these situations” (p. 41). English language learners may well have had fewer opportunities to hear and use this language than native English speakers, and so you, as teacher, might want to be especially attentive to the language that you use and elicit, in order to make sure that it is varied enough, and used in context, to build understanding.

### Two-Step Word Problems

According to NCTM (2011), there are “at least five levels of difficulty for two-step word problems” (p. 58). A two-step word problem is created by combining two subtypes from the three main types of problems described above: Add To/Take From, Put Together/Take Apart, and Compare. The chart below briefly introduces the five levels of two-step word problems—including types and subtypes—and is followed by greater detail on the topic.

5 LEVELS OF DIFFICULTY: TWO-STEP WORD PROBLEMS	ADD/SUBTRACT SITUATION TYPE	SUBTYPES
1. Two “easier” subtypes with the same operation	Add To/Take From Put Together/Take Apart	Result Unknown Total Unknown
2. Two “easier” subtypes with opposite operations	Add To/Take From Put Together/Take Apart	Result Unknown Total Unknown
3. One “easier” subtype and one “medium” subtype	Add To/Take From Put Together/Take Apart Compare: with language that <i>suggests the operation</i> needed to solve the problem	Change Unknown Addend Unknown Difference Unknown Bigger Unknown Smaller Unknown
4. Two “medium” subtypes	Add To/Take From Put Together/Take Apart Compare: with language that suggests the operation needed to solve the problem	Change Unknown Addend Unknown Difference Unknown Bigger Unknown Smaller Unknown
5. One “difficult” subtype combined with any other subtype.	Add To/Take From Compare: with language that suggests the <i>incorrect</i> operation	Start Unknown Difference Unknown Bigger Unknown Smaller Unknown

### 1. Two easier subtypes with the *same* operation

Add To/Take From—Result Unknown and Put Together/Take Apart—Total Unknown are both easier subtypes. Level 1 problems include these subtypes such that the operations required are either both addition or both subtraction. For example, “There were 28 books on a shelf in the library at the start of the school day. 12 books *from that shelf* were checked out in the morning. Eight books *from that shelf* were checked out in the afternoon. How many books remained on the shelf at the end of the day?” Here, students might first subtract 12 books from the starting total of 28, and then subtract 8 more books to find the remaining amount. Some students will approach it differently, *adding* 12 and 8 to see how many books were removed, and then subtracting that result from 28. Though that is a “level 2” approach (see next level) some students find it easier.

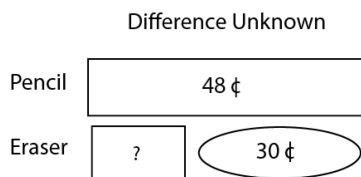
### 2. Two easier subtypes with *opposite* operations

Some problems *explicitly* call for using different operations in the two steps. For example, “At recess, 21 students were playing ball. Eight students joined them. Three students left to get a drink of water. How many children were left playing ball?” Here, the problem solver adds the 8 students who joined and subtracts the 3 students who left to drink water.

### 3. One easier subtype and one medium subtype (where students are solving for an unknown addend)

A medium subtype may include any unknown addend subtype (Change Unknown, Addend Unknown) or any of the three kinds of Compare problems, presented in language that suggests the appropriate operation. For example: “A pencil costs 48 cents and an eraser costs 30 cents less. How much do a

pencil and an eraser cost together?” Here the student first solves to find the cost of an eraser by thinking about the unknown difference.



Then the student finds the total cost of the pencil and the eraser by adding the two costs (48 cents + 18 cents = 66 cents). This second step, finding the total is considered an easy subtype (Put Together/Take Apart—Result Unknown).

**4. Two medium subtypes** (each presented in language that suggests the appropriate operation)

An example of this level of difficulty is: “Catie had eight fireflies in a jar. One night she caught some more but forgot to count how many. The next night she only caught four. If she now has 28 fireflies in her jar, how many did she catch the first night?” The first step is a Change Unknown problem (she started with 8 and ended with 28, so  $8 + \underline{\hspace{1cm}} = 28$ ). Once the 20 is known, the problem becomes an Addend Unknown problem (she caught 20 new fireflies, an unknown number on the first night and 4 on the second,  $\underline{\hspace{1cm}} + 4 = 20$ ). Both of these are medium difficulty, making this a level 4 difficulty.

**5. One difficult subtype combined with any other subtype**

Difficult subtypes include any Start Unknown problems or Compare problems involving language that does not suggest the required operation, or misleading language that suggests the wrong operation (NCTM, 2011). An example of this difficulty level: “James had some jellybeans in a jar. He gave 18 to Jenny and 23 more to Jamal. He counted 25 jellybeans left in his jar. How many did he have to start with?” In this problem, the student encounters two Start Unknown problems. One way to approach this is to begin with the final total of 25, then calculate how many James had before he gave 23 to Jamal, then calculate how many he had before he gave 18 to Jenny. These problems are both difficult problems because they are Start Unknown. The words “more” and “left” can be misleading.

In grade 2, it is likely that most of your students will focus on solving two-step word problems at the first three levels (NCTM, 2011), but you may find that you have some students who are ready for the fourth and fifth. These last two levels are included in Story Cards—Set C.

All word problems, especially two-step problems, involve additional layers of cognitive load: reading the problem, making sense of its context and question, representing the situation, and finding the solution. The language load is especially taxing for ELL students or weak readers, and if their performance on word problems is lower than you’d like, it may have nothing to do with mathematical capability. You may find that some students require the story problem to be read to them or may need physical objects (such as counters or base-10 blocks) in place of a drawing to represent the problem even if they would have no such need just to perform the same calculations. These are certainly allowed, but the need should be noted. Also keep in mind that it is a grade-2 goal that students be able to represent these situations using drawings and equations, so it is important to help students move towards that goal.

It also may help students to learn to talk themselves through two-step problems, asking themselves, for example, “What is the first question that I should answer?” or even to start by *ignoring* the final question (for the moment) and asking themselves “What *can* I figure out from this?” and then, “Does

that help me know what to do next?" Consider the pencil/eraser problem on the previous page. The problem may be easier to solve for some students if they first ask and answer the question, "How much does the eraser cost?" This strategy may help some students in solving two-step word problems and could aid when explaining their solutions to others.

## Student Variability in Representing and Solving

By grade 2, students are working on varying levels of representing and solving word problems (NCTM, 2011). Students think in different ways about a problem situation and how to represent it. Some still benefit from representing the situation with physical objects or drawings; others may write an equation or move directly to a solution, having done the arithmetic mentally (NCTM, 2011, p.54). It can be useful to note *where* each individual is in his or her own understanding and style.

Here are two examples of student work that illustrate the different approaches that students may take to the following level-3, two-step word problem.

Lee went to the zoo and saw 37 animals.  
She saw 19 monkeys, 14 birds, and some tigers.  
How many tigers did Lee see at the zoo?

The first example found below shows that a student has written an equation to represent the first step of adding, but also represented the problem with a drawing.

$19 + 14 = 33$  animals  
 m b label  
 $10 + 23 = 33$   
 label  
 $33 + 4 = 37$  animals in all  
 label

In this drawing, the student first drew 19 as a ten-stick and 9 dots and then drew 14 as a ten-stick and 4 dots, and then regrouped, combining the original 9 dots with one more from the 4 dots to make another 10. In the second step, the student figured out how many more tigers were needed to equal the total of 37 animals. Here the student did not make a drawing, instead opting to solve this step with an equation. From the written record alone, we couldn't say whether the drawing was needed, or included only to "explain" to the reader why the arithmetic in the first line is correct. Observation would tell us.

The second example includes only the solution computation. Assuming that the paper recorded everything (that no objects were used), this shows that she was able to visualize the situation and operate mentally without any additional support for both steps. Otherwise, her reasoning was the same as in the first example. This one *looks* more advanced, but they may both be equally adept at solving problems of this kind.

$$\begin{array}{r}
 19 \text{ monkeys} \\
 + 14 \text{ birds} \\
 \hline
 33 \text{ animals}
 \end{array}
 \rightarrow
 \begin{array}{r}
 33 \\
 + \boxed{4} \text{ tigers} \\
 \hline
 37 \text{ animals}
 \end{array}$$

You may find that some of your students will use a creative combination of objects, drawings, and equations when solving a problem (NCTM, 2011). This may be especially true for situations involving one easy and one medium subtype (level 3), such as Compare problems or Put Together/Take Apart—Addend Unknown problems. Any combination of drawings or equations should certainly be allowed. For second graders, it is often challenging to represent some two-step word problems using a single equation (NCTM, 2011). In fact, it’s unlikely that they have even seen such equations, so this should not be forced; instead, you will want to observe if and how students approach solving the problem at hand.

Another area in which students’ understanding varies is in the importance of labeling. Labeling relates students’ thinking back to the problem situation, putting it into context (MP2). Also, labeling is helpful to students when explaining their thinking and for seeing the connection between different classmates’ representations and methods for solving a problem. In doing so, students develop their reasoning and their attention to precision (MP6). The first example above is partly labeled; the second is fully labeled.

The main purpose of solving *any* word problems is to give students practice making sense and solving problems that arise in *the real world*. Even young students need opportunities to gather information, make sense of a problem, consider various approaches, be flexible, have stamina, solve, and check the reasonableness of their thinking. This is the essence of MP1—the ability to make sense of problems and persevere in solving them. In the context of school, word problems give students this opportunity, but it is always helpful to keep in mind the broader implications for this work grounded in *everyday life*.

### Standards for Mathematical Practice

Students build their mathematical habits of mind around two Standards for Mathematical Practice during this task: *MP1: Make sense of problems and persevere in solving them* and *MP2: Reason abstractly and quantitatively*. Solving various types of two-step word problems directly builds students’ understandings of how to make sense of a problem and persevere (MP1)—even when progress feels hard. This task provides students with multiple opportunities to think about “the meaning of a problem” and “look for entry points” to its solution (NGA & CCSSO, 2010). Thus, students continually think about whether their approach makes sense, given the context of the problem at hand. They evaluate whether or not their strategies—and their solution—is reasonable. Through these experiences, students develop their stamina and flexibility. They learn to keep going and persist even if the problem is challenging, but also understand that there are instances when trying an alternative approach may be beneficial when it appears that the path they are on is not working.

Students build their understanding of how to reason abstractly and quantitatively (MP2) when they “decontextualize” and “contextualize” an addition or subtraction situation. According to the *Common Core State Standards for Mathematics* (NGA & CCSSO, 2010), to decontextualize a situation involves representing a given situation symbolically. “Contextualizing” or “re-contextualizing” involves putting the answer in context with a label (NGA & CCSSO, 2010), for example, knowing that the answer 21 is not just 21, but 21 animals. In this task, students are solving various types of two-step word problems and

are asked to represent each problem with objects, drawings, or equations. As students move towards using equations, they gain experience in reasoning abstractly and quantitatively.

### For More Information

- Cross, C. T., Woods, T. A., & Schweingruber, H. (Eds). (2009). Committee on Earth Childhood Mathematics, Center for Education, Division of Behavioral and Social Science and Education & National Research Council. (2009). *Mathematics learning in early childhood: Paths toward excellence and equity*. Washington, DC: National Academies Press.
- National Council of Teachers of Mathematics (NCTM). (2009). *Focus in grade 1: Teaching with curriculum focal points*. Reston, VA: author.
- National Council of Teachers of Mathematics (NCTM). (2011). *Focus in grade 2: Teaching with curriculum focal points*. Reston, VA: Author.
- National Governors Association Center for Best Practices, & Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. Washington, DC: Authors. Retrieved from [http://www.corestandards.org/assets/CCSSI\\_Math%20Standards.pdf](http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf)

## GET SET: Prepare to Introduce the Task

1. Gather the materials listed on page 1.

### Part 1

Make 1 copy of Story Cards—Set A for each pair of students. Cut cards ahead of time.

### Part 2

Make 1 copy of Story Cards—Set B for each pair of students. Copy 1 set of Story Cards—Set C and 1 set of Set D (Set D is for extension only), as needed, 1 per pair of students, as they progress through the task.

2. Pair students ahead of time as partners. Model the activity to the whole class or a small group to start. To begin, invite two students to sit together. They will need a writing surface, such as a table or clipboard. Once the activity has been modeled, the whole class can play simultaneously in pairs. Observe pairs as you feel it is most useful; the GO section “Observations of Students” column will help.

### Introducing the Task

The task begins with an activity, which progresses in two parts. Throughout this document, when specific language is suggested, it is shown in *italics*.

First shuffle the Story Cards (Set A, B or C) and place a set face down in a deck in front of each player.

1. To both players: *Today you will be math book writers, writing some word problems for others to solve. You'll have a chance to solve some problems, too, and talk with a partner about how you solved each problem.*
2. *Now, each of you, choose one Story Card and turn it face up. What do you notice about the Story Cards?* (Students should describe the blanks that ask them to fill in missing information such as names or numbers needed in the story.)



3. To both players: *If you have ever played Mad Libs, these cards are just like those stories. Fill in the blanks with the missing information described below the blank.*
4. To both players, after both have completed their stories: *Read your story aloud to your partner.*
5. To both players: *First, solve your own problem.*
6. To both players: *Then, trade so that you can solve the problem your partner wrote. (jokingly) Don't cheat by looking at your partner's work first. Do it by yourself.*
7. To Player 1: *Now share your strategies for solving your problem.* To Player 2: *Say whether you agree or disagree with the solution and explain your own strategy.*
8. To Player 2: After Player 1 has shared: *Now let's do the same with your problem. You share your answer and strategies, and (turning to player 1) you will say whether you agree or disagree with the solution and say what strategies you used.*
9. *Once you have completed one round, raise your hand and you'll do the same thing with a new Story Card. Your goal is to make up and solve three problems each.*

### **Preparing to Gather Observation Data and Determine Next Steps in Instruction**

As students engage in the task, the notes in the next section will help you identify students' current strengths and possible next steps for instruction. As you observe, use whichever form of the Observation Checklist that best helps you record your observations of students and other relevant evidence as you see it: Individual, Partner, or Class. These varied forms, available at the end of this document and in a separate MS Excel file, are intended to give you a choice about how to collect notes on your students and determine possible next steps for instruction.

### **Addressing Student Misconceptions/Errors**

As students solve these two-step addition and subtraction situations with numbers less than 100, there are a few common student challenges that you may observe.

- Some Compare situations are misleading because they use a word within the context of the story that suggests the opposite of the action needed for the solution (NCTM, 2009), such as saying "fewer" when you need to add or saying "more" when you need to subtract. For example:

*Johnny has 37 pet elephants.*

*He has 21 fewer elephants than Matt, but 11 more elephants than Maria.*

*How many elephants does Matt have?*

Students often overgeneralize the word "fewer" to mean to subtract. However, to find the number of elephants Matt has, we must *add* 21 to 37, not subtract as the language might suggest. Be mindful of these types of comparison situations and take notice of how your students interpret the language.

- Understanding language is critically important to being able to solve a one-step word problem, let alone a two-step problem, which is inherently more complex. You may observe that some of your students are still developing their comprehension strategies and understanding of mathematical vocabulary. Also, questions used in word problems can vary but hold the same meaning. For example: Jessica and Justin have the same total number of marbles. Jessica has 14 blue marbles and 18 white marbles. Justin has 15 blue marbles and some white marbles." One question that could be asked is, "How many white marbles does Justin have?" A different question that could be asked—but one with the *same intended meaning*— is "How many white marbles does Justin need so that he has the same number as Jessica?" The difference in syntax and sentence structure may seem minor to



adults, but can matter greatly to students. That second question is considerably harder to read and understand. Pay attention to how your students respond. Are there specific words, phrases, or questions that students understand with ease? Are there others that are more challenging? Identifying the answers to these questions will help you uncover misconceptions or misunderstandings of language.

- Students may, of course, generate incorrect answers. Pay careful attention to *which* strategies students use and *how* they use them. Is the incorrect answer caused by a counting error? A student's misunderstanding of the operation of addition or subtraction? A developing understanding of language? Asking students to explain their strategies can help you pinpoint why the error occurred, which can help you decide how best to help your students.

### **Extensions and Elaborations**

Story Cards—Set D, to be used for Extension only, includes two-step word problems where students are prompted to write an equation with a variable for an unknown quantity, preparing students for standard 3.OA.D.8. For example, if presented with the problem, “Lorena bought 60 balloons for her brother’s birthday party: 26 were red, 18 were blue, and the rest were yellow. Write an equation that matches this story using a variable to represent the unknown number,” a student could write  $26 + 18 + y = 60$ . Take note if students are able to generalize their understanding of solving different addition and subtraction situations and extend it to using a variable to stand for an unknown quantity. You might also choose to have students solve the problems in Set D after they represent the story with an equation, depending on student readiness.

## GO: Carry Out the Task

Task Steps	Keep in Mind	Observations of Students
<p>1. Shuffle Story Cards—Set A, B, or C depending on whether you are using Part 1 or Part 2 of the task. Place the cards face-down in a deck.</p> <p>SAY to BOTH PLAYERS:</p> <p><i>Today you will be math book writers, writing some word problems for others to solve. You'll have a chance to solve some problems, too, and talk with a partner about how you solved each problem.</i></p> <p>2. Now, each of you, choose one Story Card and turn it face up. What do you notice about the Story Cards?</p> <p>Students should describe the blanks that ask them to fill in missing information such as names or numbers needed in the story.</p>	<ul style="list-style-type: none"> <li>Part 1 uses Story Cards—Set A, which includes two-step word problems (levels 1 to 3) with single-digit numbers and teen totals <math>\leq 18</math>.</li> <li>Part 2 uses Story Cards—Set B, which includes two-step word problems (levels 1 to 3) with totals from 20 through 100. Story Cards—Set C includes two-step word problems (levels 3 to 5) with totals from 20 through 100.</li> <li>Do students observe that pieces of information are missing (blank spaces)?</li> <li>Do students notice that each blank space represents a different part of the story (e.g., characters' names, setting, numbers representing quantities)?</li> </ul>	<p>A. Student makes no relevant observations about the Story Card.</p> <p>B. Student independently observes that there are missing pieces of information (blanks) in each story—all representing different parts of the story (e.g., characters' names, setting, numbers representing quantities).</p>
<p>3. Prompt both players to fill in the missing information on their Story Cards.</p> <p>SAY to BOTH PLAYERS:</p> <p><i>If you have ever played Mad Libs, these cards are just like those stories. Fill in the blanks with the missing information described below the blank.</i></p>	<ul style="list-style-type: none"> <li>How do students go about filling in the blanks of their word problems? <ul style="list-style-type: none"> <li>Are students able to generate words for missing information (e.g., if the blank calls for a name of a boy, the student writes a boy's name)?</li> <li>Are students able to accurately fill in missing numbers given the constraints of the blank?</li> </ul> </li> <li>Do students' word problems make sense given the constraints set in the blanks?</li> </ul>	<p>C. Student makes little to no attempt to correctly fill in the blank spaces of the word problem.</p> <p>D. Student attempts to fill in the blank spaces of the word problem, but at least one missing piece of information does not correctly match the constraints set in the blank space.</p> <p>E. Student's word problem makes sense given the constraints set in the blank spaces.</p> <p>F. Student appeals for teacher support—either with filling in a missing piece of information</p>

Task Steps	Keep in Mind	Observations of Students
		or reading the story aloud.
<p>4. After both players have completed their stories,</p> <p>SAY to BOTH PLAYERS:</p> <p><i>Read your story aloud to your partner.</i></p> <p>5. SAY to BOTH PLAYERS:</p> <p><i>First, solve your own problem. There is space on each story card to show your work.</i></p>	<ul style="list-style-type: none"> <li>Do students require the use of physical objects (e.g., counters or base-10 blocks) to represent the situations? If so, how do students use them?</li> <li>Do students create a drawing to represent the situation? If so, do the drawings accurately match the context of the stories?</li> <li>Do student attempt to solve the problem by writing equations? If so, does each equation correctly match the situation?</li> <li>Do students use a combination of physical objects, drawings, or equations to solve the problem? Do they label their drawings or totals?</li> <li>Do students solve one step of the problem correctly but make an error in solving the other step? If so, what is the cause of the error?</li> </ul>	<p>G. Student uses physical objects to represent and solve the problem.</p> <p>H. Student makes drawings to represent and solve the problem.</p> <p>I. Student writes equations to represent and solve the problem.</p> <p>J. Student uses a combination of physical objects, drawings, or equations to represent and solve the problem.</p> <p>K. Student correctly labels his or her drawing or work.</p> <p>L. Student does not calculate either step of the twoy step problem correctly.</p> <p>M. Student calculates one step of the problem correctly, but makes an error in solving the other step.</p> <p>N. Student calculates both steps of the problem correctly.</p>
<p>6. Once both players have finished solving their problems, ask them to trade. Each player should now work on solving their partner's problem independently.</p> <p>SAY to BOTH PLAYERS:</p> <p><i>Trade so that your partner has a chance to solve your problem. (jokingly) Don't cheat by looking at your partner's work first. Do it by yourself.</i></p>	<p>Note: See "Keep in Mind" notes above for steps 4 and 5.</p>	<p><b>Note:</b> Observation points G–N are repeated from above because it is a second chance to observe them.</p>

Task Steps	Keep in Mind	Observations of Students
<p>7. Once both players have finished solving their partner's problem, say that they can now share their solutions and strategies and explain why they do or don't agree with their partner's thinking.</p> <p>SAY to PLAYER 1:</p> <p><i>Explain how you solved your story problem. What strategies did you use? Tell us about your thinking.</i></p> <p>Once Player 1 has shared, ask Player 2 to agree or disagree with Player 1's thinking and explain why.</p> <p>SAY to PLAYER 2:</p> <p><i>Say whether you agree or disagree with the solution and explain your own strategy. How did you solve this story problem? Did you arrive at the same total?</i></p> <p>8. Switch roles so that Player 2 shares his or her explanation, and Player 1 responds by either agreeing or disagreeing and then sharing his or her explanation.</p>	<ul style="list-style-type: none"> <li>Some students will benefit from having access to sentence starters to provide language support to their justifications. It may be helpful to have one or both of the following available to students, posted in a visible location (e.g., white board, sentence strip): <ul style="list-style-type: none"> <li>I solved the first step of my problem by...</li> <li>I solved the second step of my problem by...</li> </ul> </li> <li>How do students explain how they solved their word problems?</li> <li>Do students provide an explanation for each step of the problem?</li> <li>How do students respond to the other player?</li> <li>Do students correct any errors on the part of their partners? If so, do students explain their thinking in a clear manner, so that their partners understand their errors?</li> <li>Do students benefit from the use of a sentence starter? <ul style="list-style-type: none"> <li><i>Your strategy for solving the problem makes sense (or does not make sense) because...</i></li> <li><i>I agree (disagree) with how you solved the problem because...</i></li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>Student provides little to no explanation for the reasoning used to solve his or her word problem, even with the support of a sentence frame.</li> <li>Student attempts to explain his or her reasoning and provide a justification for the rationale used. However, the justification is often incomplete or flawed.</li> <li>Student is able to explain his or her reasoning and provide a justification for the rationale used. Student's explanation is thorough and complete. Student requires no additional support (e.g., sentence starters) when responding.</li> </ul>

Task Steps	Keep in Mind	Observations of Students
<p>9. Once both players have had the opportunity to respond to one another, the round is over. The activity continues with each player choosing a new Story Card and repeating the steps of the task. Each player should create and solve 3 stories/problems.</p>		

**OBSERVATION CHECKLIST**

## ASSESSING STUDENT UNDERSTANDING: SOLVING TWO-STEP WORD PROBLEMS

Use this page to record individual student observations. Use the letters to notate each event as you see it unfold. This record is intended to help you plan next steps in your instruction for your students.

Student Name	Observations of Student	Possible Individual Student Observations	
		<b>MAKING MEANING</b> A. Student makes no relevant observations about the Story Card. B. Student independently observes that there are missing pieces of information (blanks) in each story—all representing different parts of the story (e.g., characters' names, setting, numbers representing quantities). C. Student makes little to no attempt to correctly fill in the blank spaces of the word problem. D. Student attempts to fill in the blank spaces of the word problem, but at least one missing piece of information does not correctly match the constraints set in the blank space. E. Student's word problem makes sense given the constraints set in the blank spaces. F. Student appeals for teacher support—either with filling in a missing piece of information or reading the story aloud.	<b>REPRESENTATIONS</b> G. Student uses physical objects to represent and solve the problem. H. Student makes drawings to represent and solve the problem. I. Student writes equations to represent and solve the problem. J. Student uses a combination of physical objects, drawings, or equations to represent and solve the problem. K. Student correctly labels his/her drawing or work. L. Student does not calculate either step of the twoO step problem correctly. M. Student calculates one step of the problem correctly, but makes an error in solving the other step. N. Student calculates both steps of the problem correctly.
			<b>EXPLAINING REASONING</b> O. Student provides little to no explanation for the reasoning used to solve his word problem. P. Student attempts to explain her reasoning and provide a justification for the rationale used. However, the justification is often incomplete or flawed. Q. Student is able to explain her reasoning and provide a justification for the rationale used. Student's explanation is thorough and complete. Student requires no additional support (e.g., sentence starters) when responding.

Name: \_\_\_\_\_ Story Card - A1

\_\_\_\_\_ had 18 pencils in his desk at the start  
(boy's name)  
of the school day.

Then \_\_\_\_\_ broke during Writing and  
(number < 6)

\_\_\_\_\_ more broke during Math.  
(number < 6)

How many unbroken pencils were left after Math class?

Name: \_\_\_\_\_ Story Card - A2

\_\_\_\_\_ went to the store and bought  
(girl's name)

\_\_\_\_\_ apples and \_\_\_\_\_ bananas. Then she was  
(5 < number < 10) (number < 5)

still hungry so she bought \_\_\_\_\_ oranges.  
(number < 5)

How many pieces of fruit did she buy in all?

Cut

Name: \_\_\_\_\_ Story Card - A3

\_\_\_\_\_ has 8 red marbles and \_\_\_\_\_  
(girl's name) (5 < number < 10)

blue marbles in a bag.

She took 4 blue marbles out of the bag.

How many marbles are left in \_\_\_\_\_'s bag?  
(same girl's name)

Name: \_\_\_\_\_ Story Card - A4

9 kids were playing \_\_\_\_\_ at recess.  
(name of recess game)

\_\_\_\_\_ kids left to play on the slide, but then  
(number < 5)

\_\_\_\_\_ other kids decided to join in the game.  
(5 < number < 10)

How many kids are now playing \_\_\_\_\_?  
(name of recess game)

Cut



Name: \_\_\_\_\_ Story Card - A5

\_\_\_\_\_ has 11 oranges and \_\_\_\_\_ pears.  
(girl's name) (5 < number < 10)

\_\_\_\_\_ has a total of 8 oranges and pears.  
(boy's name)

How many more oranges and pears does \_\_\_\_\_  
(same girl's name)

have than \_\_\_\_\_?  
(same boy's name)

Cut

Name: \_\_\_\_\_ Story Card - A6

\_\_\_\_\_ has 9 purple flowers and \_\_\_\_\_  
(boy's name) (5 < number < 10)

yellow flowers.

\_\_\_\_\_ has 5 fewer flowers than \_\_\_\_\_.  
(girl's name) (same boy's name)

How many flowers does \_\_\_\_\_ have?  
(same girl's name)

Name: \_\_\_\_\_ Story Card - B1

\_\_\_\_\_ has 68 stickers in his sticker book.  
(boy's name)

Then he gave \_\_\_\_\_ stickers to \_\_\_\_\_  
(25 < n < 30) (name of a friend)

and \_\_\_\_\_ more to his sister.  
(15 < n < 20)

How many stickers were left in his sticker book?

Name: \_\_\_\_\_ Story Card - B2

\_\_\_\_\_ was building a tower with blocks. She  
(girl's name)

used 34 red blocks and \_\_\_\_\_ yellow blocks.  
(20 < n < 30)

But she still wanted to make her tower taller, so she used

\_\_\_\_\_ blue blocks. How many blocks did she use total?  
(15 < n < 20)

Cut

Name: \_\_\_\_\_ Story Card - B3

\_\_\_\_\_ has 36 erasers and \_\_\_\_\_ pencils  
(girl's name) (41 < n < 58)

in her desk. She took \_\_\_\_\_ erasers out of her desk  
(18 < n < 32)

and put them in her backpack.

What's the total number of erasers and pencils left inside

\_\_\_\_\_ 's desk?  
(same girl's name)

Name: \_\_\_\_\_ Story Card - B4

39 kids were playing \_\_\_\_\_ after school.  
(name of a sport)

\_\_\_\_\_ kids decided to go home, but  
(12 < n < 29)

then \_\_\_\_\_ other kids came and played.  
(32 < n < 41)

How many kids were still playing \_\_\_\_\_ ?  
(same sport)

Cut

Name: \_\_\_\_\_ Story Card - B5

\_\_\_\_\_ picked 35 carrots and \_\_\_\_\_  
(girl's name) (33 < n < 57)

tomatoes from her family's garden.

Her mom picked \_\_\_\_\_ carrots and tomatoes in total.  
(37 < n < 53)

How many more carrots and tomatoes did \_\_\_\_\_  
(same girl's name)

pick than her mom?

Cut

Name: \_\_\_\_\_ Story Card - B6

\_\_\_\_\_ baked \_\_\_\_\_ cupcakes and  
(boy's name) (21 < n < 43)

\_\_\_\_\_ cookies. \_\_\_\_\_ baked 27 fewer  
(37 < n < 68) (girl's name)

cupcakes and cookies total than \_\_\_\_\_.  
(same boy's name)

How many cupcakes and cookies did \_\_\_\_\_ bake?  
(same girl's name)

Name: \_\_\_\_\_ Story Card - C1

\_\_\_\_\_ had \_\_\_\_\_ jellybeans in a jar. Then  
(boy's name) (84 < n < 99)

he gave some jellybeans to \_\_\_\_\_  
(name of a friend)

and \_\_\_\_\_ jellybeans to his brother. At that point,  
(21 < n < 28)

\_\_\_\_\_ counted 25 jellybeans left in his jar.  
(same boy's name)

How many jellybeans did he give to his friend?

Name: \_\_\_\_\_ Story Card - C2

On Friday night, \_\_\_\_\_ caught some fireflies  
(girl's name)

in a jar. The next night she caught \_\_\_\_\_ more fireflies .  
(26 < n < 41)

Then, on the third night, she caught \_\_\_\_\_ more fireflies.  
(22 < n < 37)

By that time, she had caught 93 fireflies in all.

How many fireflies did she catch on Friday night?

Cut

Name: \_\_\_\_\_ Story Card - C3

\_\_\_\_\_ had some coins saved in a piggy bank.  
(boy's name)

He decided to give \_\_\_\_\_ coins to \_\_\_\_\_.  
(15 < n < 20) (name of a friend)

and \_\_\_\_\_ coins to his cousin. Then \_\_\_\_\_.  
(25 < n < 30) (same boy's name)

had 31 coins left in his piggy bank.

How many coins did he have to start?

Name: \_\_\_\_\_ Story Card - C4

\_\_\_\_\_ scored \_\_\_\_\_ fewer points in the  
(boy's name) (28 < n < 37)  
basketball game than \_\_\_\_\_.  
(girl's name)

\_\_\_\_\_ scored \_\_\_\_\_ points in the first half  
(same girl's name) (35 < n < 40)

and \_\_\_\_\_ points in the second half.  
(30 < n < 35)

How many points did \_\_\_\_\_ score?  
(same boy's name)

Cut

Name: \_\_\_\_\_ Story Card - C5

\_\_\_\_\_ brought 28 slices of cheese and \_\_\_\_\_  
(girl's name) (22 < n < 31)

crackers to a friend's party. Derek brought a combo platter

with \_\_\_\_\_ slices of cheese and crackers in total.  
(73 < n < 96)

How many fewer slices of cheese and crackers did

\_\_\_\_\_ bring to the party than Derek?  
(same girl's name)

Name: \_\_\_\_\_ Story Card - C6

\_\_\_\_\_ baked \_\_\_\_\_ muffins and  
(girl's name) (29 < n < 39)

\_\_\_\_\_ brownies. This was \_\_\_\_\_ fewer  
(27 < n < 37) (13 < n < 24)

muffins and brownies than \_\_\_\_\_ baked.  
(boy's name)

How many muffins and brownies did \_\_\_\_\_  
(same boy's name)

bake in all?

Cut

Name: \_\_\_\_\_ Story Card - D1

\_\_\_\_\_ had 35 pencils in his desk at the start  
(boy's name)  
of the school day.

Then \_\_\_\_\_ lost some during Writing and lost  
(same boy's name)  
13 more during Social Studies. By the end of the day,  
he only had \_\_\_\_\_ pencils left in his desk.  
( $15 < n < 20$ )

Write an equation for the story using a variable to  
represent the unknown number.

Name: \_\_\_\_\_ Story Card - D2

\_\_\_\_\_ went to the store and bought  
(girl's name)

\_\_\_\_\_ carrots and \_\_\_\_\_ cucumbers. Then she  
( $10 < n < 15$ ) ( $15 < n < 20$ )

was still hungry so she bought some peppers.

\_\_\_\_\_ bought 42 vegetables in all.  
(girl's name)

Write an equation for the story using a variable to  
represent the unknown number.

Cut



Name: \_\_\_\_\_ Story Card - D3

\_\_\_\_\_ had some baseball cards in a box.  
(boy's name)

He bought \_\_\_\_\_ more baseball cards at the store  
( $15 < n < 20$ )

and got \_\_\_\_\_ more from his cousin. By then,  
( $45 < n < 55$ )

\_\_\_\_\_ had 78 baseball cards in all.  
(same boy's name)

Write an equation for the story using a variable to represent the unknown number.

Cut

Name: \_\_\_\_\_ Story Card - D4

\_\_\_\_\_ scored \_\_\_\_\_ goals in the  
(boy's name) ( $5 < n < 10$ )

soccer game. \_\_\_\_\_ scored some goals  
(girl's name)

in the first half and \_\_\_\_\_ goals in the second half.  
( $6 < n < 11$ )

Together, they scored 21 goals in all.

Write an equation for the story using a variable to represent the unknown number.

Name: \_\_\_\_\_ Story Card - D5

\_\_\_\_\_ bought \_\_\_\_\_ balloons for her brother's  
(girl's name) (55 < n < 60)

birthday party. Some balloons were red, \_\_\_\_\_ were blue  
(15 < n < 20)

and \_\_\_\_\_ were yellow.  
(20 < n < 25)

Write an equation to match the story using a variable to represent the unknown number.

Name: \_\_\_\_\_ Story Card - D6

\_\_\_\_\_ baked \_\_\_\_\_ brownies and  
(boy's name) (45 < n < 58)

cookies. \_\_\_\_\_ baked \_\_\_\_\_  
(girl's name) (23 < n < 32)

cookies. They sold some at a bake sale and had \_\_\_\_\_  
(13 < n < 31)

left to share with their friends.

Write an equation to match the story using a variable to represent the unknown number.

Cut